Algerian Journal of Research and Technology
https://doi.org/10.58681/ajrt.22060106
A.J.R.T Volume 6 (N°1) (2022) 48 – 57
Selection and peer-review under responsibility of the scientific committee of The 1st International Symposium on INDUSTRIAL ENGINEERING, MAINTENANCE AND SAFETY IEMS'22.

Solving a Multi-product Lot-Sizing Problem with Multi-trip Direct Shipment

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Published. April 15, 2022.

DOI: https://doi.org/10.58681/ajrt.22060106

Abstract. Although the sequential optimization of production, storage, and distribution activities in the supply chain has been studied in depth to generate significant profits. The integrated optimization of the different functions in a coherent way is essential to make a difference in the current economic context, which is experiencing fierce competition in most industrial sectors. In this paper, our study is inspired and motivated by a pharmaceutical company that owns a manufacturing center with two production units. Each production unit specializing in the production of one type of product with different rates depends on its speed, and a fleet of homogeneous vehicles is used to deliver the warehouse (which is itself the distribution center) indirect shipment. We have to take into account certain characteristics, such as the capacity of the production units, multiple products, inventory levels, security stock, and delivery requirements including vehicle capacity and several vehicles available. Each vehicle can perform multiple trips. The aim is to minimize the total cost of production, inventory, and transport. We offer a full linear mixed program for multi-product lot-sizing problems with the multi-trip direct shipment. The computational experiment results are also presented and discussed through the different scenarios.

Keywords. Integrated optimization, Lot sizing problem, Direct shipment, Multi-trip, Mixed integer programming.

INTRODUCTION

The customer-supplier connection has advanced in the current financial environment, defining the demand for product and service customization, reducing delivery delays, expanding delivery systems, and raising customer satisfaction rates. As a result, industrial companies are seeking new ways to increase efficiency and better meet customer demands. To achieve these objectives, these companies will need to execute new planning across their supply chain network.

Furthermore, changes in the supply chain might result in significant cost savings for a corporation. Companies commonly handle distinct activities based on their upstream operations in traditional supply chain systems. One of the most important optimization problems in supply chain management is an integrated optimization approach to logistic systems. A new trend in operations research is to combine and coordinate diverse planning problems to obtain better plans, balancing the benefits and drawbacks of a much larger set of options.

In the integrated production and direct shipment distribution, the products are directly transported from the manufacturing plant to the customers. The production, setup, inventory, and direct shipment costs are minimized over the planning horizon. This problem typically incorporates various production aspects, e.g., production setup cost and/or setup time, and involves distribution decisions where the fixed and unit costs of delivery are customer-specific (Adulyasak et al.,2014).

In a distribution system, vehicles are not always employed since they are allowed to make many trips. It means that a single vehicle can directly deliver to a retailer multiple times in the same period to satisfy its demand, shown in figure (1. a), and can visit the first customer on the first trip then customer two on the second trip shown in figure(1. b), and so on. The aim is to minimize the utilization cost of vehicles. Under the title Vehicle Routing Problem with Multiple Use of Vehicles, Fleischmann (1990) published the first visible attempt to handle a vehicle routing problem with multiple journeys, in the framework of resolving a slew of distribution issues involving a heterogeneous fleet of vehicles and time windows. So is Cattaruzza et al. (2016) give a clear overview of the work devoted to this problem and highlight how the possibility to link up several trips for a vehicle affects solution methods.

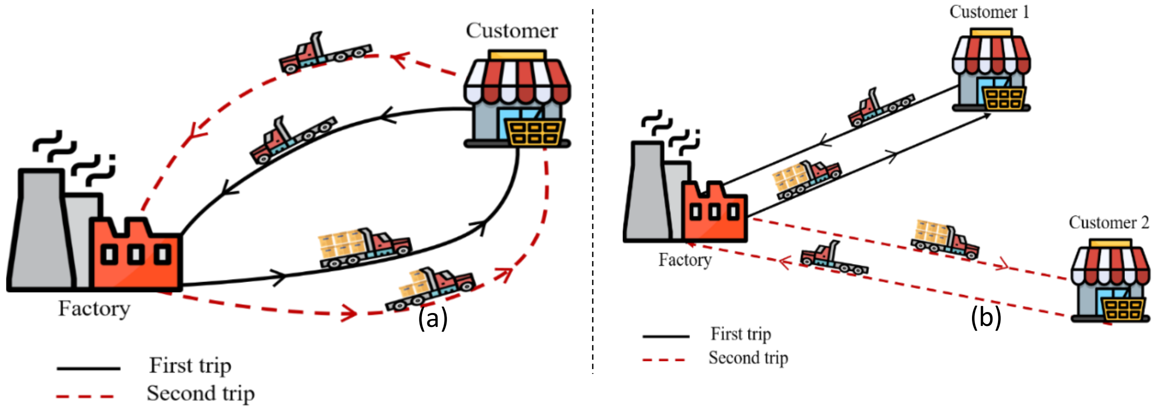

Fig.1. Represent the entire concept of transportation with the multi-trip direct shipment, (a) for a single customer; (b) for many MMMMMMM.

Literature Review

Various researchers studied this problem as Li et al. (2004) concentrated on the lot-sizing problem in which the provider can deliver by direct delivery using truckload (TL) or less-than-truckload (LTL) transportation. In the first model, they recommended that production be limited to a multiple of constant batch size in each period as well as, backlogging is permitted and all cost factors are time variable. Then an algorithm is constructed to solve the second model, which contains a generic form of product acquisition cost structure, comprising a fixed charge for each acquisition, a variable unit manufacturing cost, and a freight cost with a truckload discount, using the findings produced for the first model. Van Norden and Vande Velde (2005) investigate a multi-product lot-sizing model based on a situation with a major European manufacturing corporation, in which any portion of a reserved transportation capacity can be employed at any time in exchange for a guaranteed price. If capacity is insufficient, the shipper must contract for extra transportation capacity on the spot market, where the price is greater. As a result, the freight prices in our model are linearly growing piece by piece, then a Lagrangean relaxation algorithm to compute lower and upper bounds was proposed. Rizk et al. (2006) used a Lagrangian relaxation technique to get lower and upper bounds by decomposing the integrated issue into uncapacitated lot-sizing and time-independent subproblems, after that, a heuristic strategy based on sub-gradient optimization was given to handle a common problem in consumer products wholesaling and retailing. To overcome the problem of backlogging, Chand et al. (2007) devised a dynamic programming approach, in which, they looked at a dynamic lot-sizing problem that a producer faces when supplying a single product to multiple customers. Customers are differentiated by their backorder costs and shipping costs, with a customer with a high backorder cost having a greater need for the product than a customer with a low backorder cost. To help speed up the process with TL and LTL cost structures, Jaruphongsa et al. (2007) presented various dynamic programming techniques. Jaruphongsa and Lee (2008) investigated the problem of split delivery with time window constraints and solved it using dynamic programming methods. To solve the OWMR with a single product, Solyalı and Süral (2012) proposed a new powerful formulation based on a mixed transportation and route model, then They proved that for the joint replenishment problem, where the warehouse is a cross-docking facility, the new and transportation formulations are equal. Melo and Wolsey (2012) addressed many formulations for the two-level production–distribution problems with capacitated production and vehicles, as well as hybrid heuristics. In addition, A different approach for a problem in a similar context is described by Fleischmann, where, the multi-trip concept in vehicle routing problem (VRP) was introduced under the name Vehicle Routing Problem with Multiple Use of Vehicles, Fleischmann (1990). They develop mathematical models that seek to link the problem of production lot scheduling with outbound shipment decisions. The goal of the optimization is to reduce a manufacturer's overall relevant expenses by distributing a group of items to numerous retailers. For simplicity, the common cycle technique is used to address the economic lot-scheduling problem while making production/distribution decisions. Saglam and Banerjee (2018) create two separate shipping scenarios: periodic full truckload (TL) peddling shipments and less than truckload (LTL) direct shipping, which are both linked to and integrated with the multiproduct batching choices. Lmariouh et al. (2019) consider a Moroccan industrial application that includes the manufacture and distribution of bottled water. They consider the production process, delivery deadlines, numerous items, and inventory levels. The goal is to keep the total cost of manufacturing, shipping, and inventory as low as possible. For a variation of the multi-vehicle, multi-product production routing issue, we suggest a mixed-integer linear program. Rakiz et al. (2021) combine several aspects related to (1) multi-level and multi-product production units, (2) multi-level and multi-product storage units, and (3) train transportation with time windows. The objective is to hierarchically satisfy deterministic demand units, and minimize total operational costs. On a large real data set gathered from our industry, we demonstrate the value of integrating decisions in terms of enhancing demand satisfaction rates, increasing resource efficiency, and lowering total operational costs by reducing the total number of switch-offs of productiontriall partners. We also investigate the cost structure and conduct a full cost sensitivity analysis. Finally, this work evaluates the upstream and downstream propagation of decisions in the logistics system and underlines the benefits in terms of flexibility offered by the global approach.

PROBLEM DESCRIPTION

We begin by defining the problem in a general way. We define the model's assumptions, and then we describe the relevant entities and the parameters that are associated with them. Following that, the constraints for capturing production, inventory, and transportation decisions, as well as the linking constraints for connecting these aspects of the problem, are presented:

1. The following are the model's assumptions.
2. Demands for products are given and deterministic.
3. All demands must be satisfied.
4. Manufactory can produce multi-product at various rates, according to the production speed.
5. Production, storage, and vehicle capacity must not be exceeded.
6. Direct shipment is used for delivery.
7. Multi-trips are allowed.
8. Fixed Utilization cost of vehicles per period.

Our formulation includes the following indices, parameters, and decision variables:

Indices

j — $Index\ of\ product, j = 1 \ldots P$

t — $Index\ of\ periods, t = 1 \ldots T$

n — $Index\ \ of\ nodes, n = 1 \ldots N$

r — $Index\ of\ trips, r = 1 \ldots R$

v — $Index\ of\ homogeneous\ vehicules, v = 1 \ldots V$

c — $Index\ of\ speeds, c = 1 \ldots K$

Parameters

Production :

$Cp_{j,t,c}$ — $Production\ cost\ for\ product\ j, at\ period\ t\ with\ rate\ c$

$Cs_{j,c,t}$ — $Setup\ cost\ for\ each\ product\ j\ with\ speed\ c\ \ at\ period\ t$

$Cap_{c,t}$ — $Production\ capacity\ with\ rate\ c\ at\ \ period\ t$

Inventory:

$h_{i,j,t}$ — $Holding\ cost\ of\ product\ j\ at\ location\ i\ atperiod\ t$

$CapSt_i$ — $Maximum\ inventory\ holding\ capacity\ at\ location\ i$

$Dem_{j,t}$ — $Demand\ of\ product\ j\ et\ period\ t$

Transport :

Ct — $Fixed\ Transportation\ cost.$

$CapC$ — $Capacity\ of\ vehicules.$

CU_t — $Fixed\ utilisation\ cost\ of\ each\ vehicule\ atperiod\ t.$

Decision Variables

Production :

$x_{j,t,c}$ — $Production\ quantity\ of\ product\ j\ produced\ at\ period\ t\ with\ rate\ c.$

$y_{j,t,c}$ — $Equal\ to\ 1\ if\ there\ is\ production\ of\ product\ j\ at\ period\ t\ with\ rate\ c, 0\ oth$

Inventory :

$I_{i,j,t}$ — $Inventory\ level\ of\ product\ j\ at\ location\ i\ at\ period\ t.$

Transport :

$v_{t,r}$ — $Number\ of\ vehicule\ sent\ to\ the\ warehouse\ in\ trip\ r\ at\ period\ t.$

$q_{j,t,r}$ — $Quantity\ of\ product\ j\ delivered\ tothe\ warehouse\ in\ trip\ r\ at\ period\ t.$

H_t Number of vehicule used at period t.

Mathematical Formulation

Objective function:

$$min \sum_{t\epsilon T}\sum_{j\epsilon P}\sum_{c\epsilon C} x_{j,t,c} * Cp_{j,t,c} + \sum_{t\epsilon T}\sum_{j\epsilon P}\sum_{c\epsilon C} y_{j,t,c} * Cs_{j,t,c} + \sum_{i\epsilon N}\sum_{j\epsilon P}\sum_{t\epsilon T} I_{i,j,t} * h_{i,j,t} + \sum_{r\epsilon R}\sum_{t\epsilon T} v_{t,r} * Ct + \sum_{t\epsilon T} CU_t * H_t \quad (1)$$

Constraints:

$$I_{1,j,t} = I_{1,j,t-1} + \sum_{c\epsilon C} x_{j,t,c} - \sum_{r\epsilon R} q_{j,t,r} \qquad \forall\, j\epsilon P\ ,t\epsilon T-\{1\} \quad (2)$$

$$I_{i,j,t} = I_{i,j,t-1} + \sum_{r\epsilon R} q_{j,t,r} - Dem_{j,t} \qquad \forall\, j\epsilon P\ ,t\epsilon T-\{1\} \quad (3)$$

$$\sum_{j\epsilon P} I_{i,j,t} \le CapSt_i \qquad \forall\, i\epsilon N\ ,t\epsilon T \quad (4)$$

$$\sum_{j\epsilon P} x_{j,t,c} \le Cap_{c,t} \qquad \forall c\epsilon K\ ,t\epsilon T \quad (5)$$

$$x_{j,t,c} \le y_{j,t,c} * \sum_{t1\in T} Dem_{j,t1} \qquad \forall j\epsilon P, c\epsilon K\ ,t\epsilon T \quad (6)$$

$$\sum_{c\epsilon C} y_{j,t,c} \le 1 \qquad \forall j\epsilon P\ ,t\epsilon T \quad (7)$$

$$\sum_{j\epsilon P} q_{j,t,r} \le CapC * v_{t,r} \qquad \forall\, r\epsilon R\ ,t\epsilon T \quad (8)$$

$$v_{t,r} \le V \qquad \forall\, r\epsilon R\ ,t\epsilon T \quad (9)$$

$$v_{t,r} \le H_t \qquad \forall\, r\epsilon R\ ,t\epsilon T \quad (10)$$

$$y_{j,m,t,c} \in \{0,1\}\ ,\ I_{i,j,t}, x_{j,m,t,c}, q_{j,t,r}, v_{t,r}, H_t \ge 0 \quad (11)$$

Objective function (1) minimize the production cost, setup cost, holding cost at supplier and retailer, as well as the transportation cost. The inventory balance at the manufacturing and the consumers are ensured by constraints (2) and(3). Constraint (4) shows that the inventory capacity is respected. Constraint (5) models the production capacity. Constraint (6) connects production and transportation choices to guarantee that the amounts produced of each product satisfy the deliveries made in each period for every retail location. Constraint (7) indicates that only one speed must be used for production. Constraint ensures that the vehicle capacity is not exceeded. Constraint (9) indicates that the number of vehicles used for each trip at each period must respect the number of vehicles available. Constraint (10) defines the number of vehicles used in each period.

Case Study

This part details the case study of an Algerian pharmaceutical company that owns a processing center where several products are processed and delivered to the warehouse. This last is itself the distribution center that delivers goods to customers. It receives client requests and sends the overall consumer request information to the production center. The production center has two production units, each one produces one type of product at two different rates depending on the speed of the unit (depending on the speed of machines or lines in it). The manufacturing plant considers a setup cost while starting production in each unit. It has two homogeneous vehicles. Processed products may be directly charged into vehicles to be delivered or they can be stored at the production plant store to be delivered in a later period. The quantity held at the plant and warehouse must not exceed each location's stock capacity. A simplified manner of the problem is depicted in figure 2.

Some additional considered constraints must be taken into account to apply the suggested approach to the case study. We add $m: indexofproductionunits, m = 1 \dots M$, then we change $Cap_{c,t}$,$Cs_{j,c,t}$, $x_{j,t,c}$ and $y_{j,t,c}$ by $Cap_{m,c,t}$,$Cs_{j,m,c,t}$, $x_{j,m,t,c}$ and $y_{j,m,t,c}$ which represent: production capacity for each production unit m with rate c at period t, setup cost of product $j$in unit $m$with rate c at period t, production quantity of product j in production unit m with rate c at period t and a binary variable equal to 1 if unit m set up to produce product j with rate c at period t, respectively. Concerning constraints, after changing objective function and constraints(2),(5),(6) and (7), we obtained :

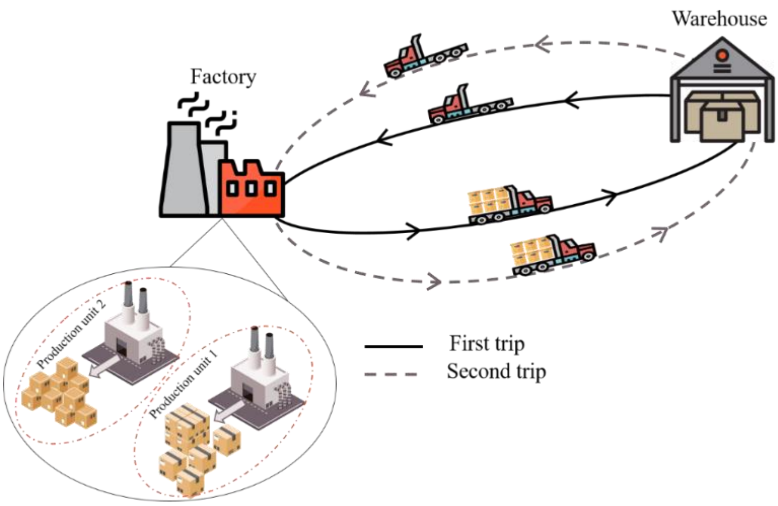

Fig.2. Representation of a simplified overview of the problem.

$$min \sum_{t\epsilon T} \sum_{j\epsilon P} \sum_{c\epsilon C} \sum_{m\epsilon M} x_{j,m,t,c} * Cp_{j,m,t,c} + \sum_{t\epsilon T} \sum_{j\epsilon P} \sum_{c\epsilon C} \sum_{m\epsilon M} y_{j,t,c} * Cs_{j,t,c} + \sum_{i\epsilon N} \sum_{j\epsilon P} \sum_{t\epsilon T} I_{i,j,t} * h_{i,j,t} + \sum_{r\epsilon R} \sum_{t\epsilon T} v_{t,r} * Ct + \sum_{t\epsilon T} CU_t * H_t \quad (12)$$

$$I_{1,j,t} = I_{1,j,t-1} + \sum_{m\epsilon M} \sum_{c\epsilon C} x_{j,m,t,c} - \sum_{r\epsilon R} q_{j,t,r} \qquad \forall\, j\epsilon P \;, \forall\, t\epsilon T - \{1\} \quad (13)$$

$$\sum_{j\epsilon P} x_{j,m,t,c} \leq Cap_{m,t,c} \qquad \forall c\epsilon K \,, m\epsilon M \;, t\epsilon T \quad (14)$$

$$x_{j,m,t,c} \leq y_{j,m,t,c} * \sum_{t1\epsilon T} Dem_{j,t1} \qquad \forall j\epsilon P, c\epsilon K, m\epsilon M \;, t\epsilon T \quad (15)$$

$$\sum_{c\epsilon C} y_{j,m,t,c} \leq 1 \qquad \forall j\epsilon P \,, m\epsilon M \,, t\epsilon T \quad (16)$$

$$x_{1,2,t,c} = 0 \qquad \forall c\epsilon K, t\epsilon T \quad (17)$$

$$x_{2,1,t,c} = 0 \qquad \forall c\epsilon K, t\epsilon T \quad (18)$$

Constraints (17) and (18) are imposed to specialize production of processing units. Mean that production unit one produces only type number one, and production unit two produces only type number two.

EXPERIMENTATIONS

In this part, to illustrate the efficacy of our formulation, we show the experimental part of our study. We provide different trails by way of two scenarios. The solver CPLEX is used to solve our mathematical formulation.

Common data are:

1. The company serves only the warehouse, so we consider it as one client.
2. The company owns two homogenous vehicles with a capacity of 500 products.
3. The company owns a store with a capacity of 500 products.
4. The company owns two production units, each one having two speeds.
5. Each production unit produces: 800 products with speed 1 “medium” ($c = 1$) and 1000 products with speed 2 ”high” ($c = 2$) in all the periods.
6. Vehicles can have up to two trips per period.
7. Fixed utilization cost of a vehicle is 100 in all the period.
8. Fixed transportation cost from the factory to the warehouse is 250.
9. The warehouse has a store with a capacity of 200 products.
10. No initial inventories at the plant and customers (equal to zero).

Table 1 shows the data for the second scenario, which includes the identical information as the first but for the demand numbers; we set the first period's amount to below and the second period's amount to be high, and so on. The cost of inventory is higher than the cost of production and setup.

Table 1. Data of the second scenario (Scenario 02)

Period				1	2	3	4
Demand			$\boldsymbol{j = 1}$	100	800	50	700
			$\boldsymbol{j = 2}$	200	1000	100	600
$\boldsymbol{Cp_{j,t,c}}$	$\boldsymbol{c = 1}$		$\boldsymbol{j = 1}$	4	4	4	4
			$\boldsymbol{j = 2}$	8	8	8	8
	$\boldsymbol{c = 2}$		$\boldsymbol{j = 1}$	9	9	9	9
			$\boldsymbol{j = 2}$	11	11	11	11
$\boldsymbol{Cs_{j,m,c,t}}$	$\boldsymbol{m = 1}$	$\boldsymbol{c = 1}$	$\boldsymbol{j = 1}$	8	15	10	7
			$\boldsymbol{j = 2}$	/	/	/	/
		$\boldsymbol{c = 2}$	$\boldsymbol{j = 1}$	10	20	15	10
			$\boldsymbol{j = 2}$	/	/	/	/
	$\boldsymbol{m = 2}$	$\boldsymbol{c = 1}$	$\boldsymbol{j = 1}$	/	/	/	/
			$\boldsymbol{j = 2}$	10	18	15	10
		$\boldsymbol{c = 2}$	$\boldsymbol{j = 1}$	/	/	/	/
			$\boldsymbol{j = 2}$	20	30	25	20
$\boldsymbol{h_{i,j,t}}$	$\boldsymbol{i = 1}$ $plant$		$\boldsymbol{j = 1}$	20	20	20	20
			$\boldsymbol{j = 2}$	30	30	30	30
			$\boldsymbol{j = 1}$	20	20	20	20
	$\boldsymbol{i = 2}$ $warehouse$		$\boldsymbol{j = 2}$	30	30	30	30

RESULTS

The results of the generated scenarios are reported in this part. The models are implemented in the CPLEX solver to get the results. Tables 3 and 4 detail these findings. Each table represents a single scenario, with the objective function value listed with the amount produced, delivered, and stored, as the number of vehicles utilized in each trip to the warehouse during each period.

DISCUSSION

The results of both scenario experiments are explained in this section.

Table 2. Results of the first scenaio (Scenario 01)

Objective function			**40605**			
Costs		**Production**	**Setup**	**Inventory**	**Transport**	**Utilization**
		37350	155	0	2500	600
	Period		**1**	**2**	**3**	**4**
$x_{j,m,t,c}$	$j=1$	**Speed used**	$c=1$	$c=1$	$c=1$	$c=1$
		Quantity	0	600	500	750
	$j=2$	**Speed used**	$c=2$	$c=2$	$c=1$	$c=1$
		Quantity	1000	850	500	700
$q_{j,t,r}$	$j=1$	**Trip**	/	1 2	2	1 2
		Quantity	0	150 450	500	250 500
	$j=2$	**Trip**	1 2	1	1	1
		Quantity	500 500	850	500	700
	$I_{i,j,t}$		0	0	0	0
	H_t		1	2	1	1
$v_{t,r}$		**Trip**	**1 2**	**1 2**	**1 2**	**1 2**
		Number of vehicles used	1 1	2 1	1 1	2 1

In scenario 1, the results presented in table 2 show that due to high demand for product type two in period 1, production of this type occurred at the second speed even though the cost of producing with the second speed was higher than the cost of producing with the first speed; so we must produce at that speed to meet the client's demand. In addition, due to the vehicle's capacity of half that amount, the delivery quantity was split into two trips. We justify our decision to utilize one car for two trips since it costs 100 less than using two vehicles for one trip, which costs 200. For period 2, demands were met by producing the same quantity requested. When we observe that inventory holding cost is almost negligible and there is no production in the first period for product type one, so we ask why we did not produce 600 products in the first period then be stored for the next period? On the other hand, delivered it then stored in the warehouse? The answer is if we produce 600 products in the first period then we stored we get as a minimum cost: 600*0.025+10 = 25 (inventory and setup costs), which is higher than 20 (setup cost for first speed). Furthermore, we are unable to make Product type 2 at the first speed since it only offers 800 products, while we require 850. Moreover, the delivery amount is 1450, and when we divide it by 500, we get three, that's why we sent two vehicles to the warehouse on the first trip and one vehicle on the second. For the following 2 periods, we have just produced what is needed for this period, then delivered in two trips.

Table 3. Results of te second scenario (Scenario 02)

Objective	**27755**

function

Costs		**Production**	**Setup**	**Inventory**	**Transport**	**Utilization**
		24800	105	0	2250	600
	Period		**1**	**2**	**3**	**4**
$x_{j,m,t,c}$	$j = 1$	**Speed used**	1	1	1	1
		Quantity	100	800	50	700
	$j = 2$	**Speed used**	1	2	1	1
		Quantity	200	1000	100	600
$q_{j,t,r}$	$j = 1$	**Trip**	1	2	1	1 2
		Quantity	100	800	50	233 467
	$j = 2$	**Trip**	1	1 2	1	1 2
		Quantity	200	800 200	100	567 33
	$I_{i,j,t}$		0	0	0	0
	H_t		1	2	1	2
$v_{t,r}$		**Trip**	1	1 2	1	1 2
		Number of vehicles used	1	2 2	1	2 1

In scenario 2, the results presented in table 3 show that there is no storage in all periods, despite low demand in the first and third periods, due to the high value of inventory holding costs in both plant and warehouse. We have low demand in the first and third periods, which can be met by producing at first speed (offering 800 goods) and then delivering by a single vehicle on a single trip because the amount is less than the vehicle's capacity. In terms of the second period, we can create the first type at the first speed, but the second type must be produced at the second speed; to meet customer requirements, two vehicles must be sent on the first and second trips (1800/500 4). For the last period, it is necessary to make both types with the first speed then delivered by two vehicles on the first trip and one on the second trip.

CONCLUSION

In this paper, a large Multi-product Lot-sizing Problem with Multi-trip direct shipment is addressed which serves to optimize in the same time production, inventory, and transportation decisions. We propose a novel mathematical formulation that allows us to manufacture at various rates, which implies that production is flexible, especially when demand is high. On the other side, we include the multi-trip idea in the direct shipment. It enables the company to meet clients' demands with several trips even if it has only one vehicle, avoiding missed sales and late penalties, as well as reducing vehicle utilization costs. We believe that in the case of a single-vehicle when multiple trips are allowed, deserves to be part of this study.

In addition, we presented a case study of an Algerian pharmaceutical company, in which we applied our approach.

Our results show that adding the concept of speeds and multi-trip is more beneficial to any company compared to the other approaches.

Finally, many factors are still not taken into account in our method, resulting in new future research such as:

1. The energy consumption by the production system especially for the high speed, we want to satisfy demands but without forgetting the energy aspect.
2. The time aspect such as production time, setup time, and transportation time. To be real, we need to consider time windows and maximum trips allowed in each period
3. Developing a heuristic or a meta-heuristic that can deal with the big instances because solvers can give an exact solution for small and medium instances.

REFERENCES

Cattaruzza, D., Absi, N., & Feillet, D. (2016). The multi-trip vehicle routing problem with time windows and release dates. *Transportation Science*, *50*(2). https://doi.org/10.1287/trsc.2015.0608

Chand, S., Hsu, V. N., Sethi, S., & Deshpande, V. (2007). A dynamic lot sizing problem with multiple customers: Customer-specific shipping and backlogging costs. *IIE Transactions (Institute of Industrial Engineers)*, *39*(11). https://doi.org/10.1080/07408170601089497

Fleischmann, B. (1990). The vehicle routing problem with multiple use of vehicles. *Fachbereich Wirtschaftswissenschaften, Universität Hamburg*.

Jaruphongsa, W., Çetinkaya, S., & Lee, C. Y. (2007). Outbound shipment mode considerations for integrated inventory and delivery lot-sizing decisions. *Operations Research Letters*, *35*(6). https://doi.org/10.1016/j.orl.2007.02.009

Jaruphongsa, W., & Lee, C. Y. (2008). Dynamic lot-sizing problem with demand time windows and container-based transportation cost. *Optimization Letters*, *2*(1). https://doi.org/10.1007/s11590-006-0042-1

Li, C. L., Hsu, V. N., & Xiao, W. Q. (2004). Dynamic lot sizing with batch ordering and truckload discounts. *Operations Research*, *52*(4). https://doi.org/10.1287/opre.1040.0121

Lmariouh, J., el Hachemi, N., Jamali, M. A., Bouami, D., & Rousseau, L. M. (2019). An integrated production and distribution problem with direct shipment: A case from Moroccan bottled-water market. *International Journal of Operational Research*, *34*(1). https://doi.org/10.1504/IJOR.2019.096942

Melo, R. A., & Wolsey, L. A. (2012). MIP formulations and heuristics for two-level production-transportation problems. *Computers and Operations Research*, *39*(11). https://doi.org/10.1016/j.cor.2012.02.011

Rakiz, A., Absi, N., & Fenies, P. (2022). An integrated production and direct shipment problem in a mining industry. *International Journal of Production Research*, *60*(21). https://doi.org/10.1080/00207543.2021.1998699

Rizk, N., Martel, A., & Ramudhin, A. (2006). A Lagrangean relaxation algorithm for multi-item lot-sizing problems with joint piecewise linear resource costs. *International Journal of Production Economics*, *102*(2). https://doi.org/10.1016/j.ijpe.2005.02.015

Sağlam, Ü., & Banerjee, A. (2018). Integrated multiproduct batch production and truck shipment scheduling under different shipping policies. *Omega (United Kingdom)*, *74*. https://doi.org/10.1016/j.omega.2017.01.007

Solyali, O., & Süral, H. (2012). The one-warehouse multi-retailer problem: Reformulation, classification, and computational results. *Annals of Operations Research*, *196*(1). https://doi.org/10.1007/s10479-011-1022-0

van Norden, L., & van de Velde, S. (2005). Multi-product lot-sizing with a transportation capacity reservation contract. *European Journal of Operational Research*, *165*(1). https://doi.org/10.1016/j.ejor.2003.04.008